

Effects of parity violation on non-gaussianity of primordial gravitational waves in Hořava-Lifshitz gravity

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In this paper, we study the effects of parity violation on non-gaussianities of primordial gravitational waves in the framework of Hořava-Lifshitz theory of gravity, in which high-order spatial derivative operators, including the ones violating parity, generically appear. By calculating the three-point correlation function, we find that the leading-order contributions to the non-gaussianities come from the usual second-order derivative operators, which produce the same bispectrum as that found in general relativity. The contributions from high-order spatial n -th derivative operators are always suppressed by a factor $(H/M_*)^{n-2}$ ($n \geq 3$), where H denotes the inflationary energy and M_* the suppression mass scale of the high-order spatial derivative operators of the theory. Therefore, the next leading-order contributions come from the three-dimensional gravitational Chern-Simons term. With some reasonable arguments, it is shown that this 3-dimensional operator is the only one that violates the parity and in the meantime has non-vanishing contributions to non-gaussianities.

I. INTRODUCTION

Primordial gravitational waves (PGWs), which are expected to be generated during inflation, have attracted a great deal of attention recently, as their detections would be the direct evidence of inflation, and more important the existence of gravitational waves in the universe. From the properties of PGWs, such as their power spectra and non-gaussianities, we can extract useful information about the theory of inflation and gravity. In particular, the PGWs produce not only the temperature anisotropy, but also a distinguishable signature in the polarization of the cosmic microwave background (CMB) [1]. Decomposing the polarization into two modes: one is curl-free, the E-mode, and the other is divergence-free, the B-mode, one finds that the B-mode pattern cannot be produced by density fluctuations. Thus, its detection would provide a unique signature for the existence of PGWs [2].

In addition, PGWs normally produce the TT, EE, BB and TE spectra of CMB, but the spectra of TB and EB vanish when the parity of the PGWs is conserved [1]. However, if the theory is chiral, the power spectra of right-hand and left-hand PGWs can have different amplitudes, and then induce non-vanishing TB and EB correlation in large scales [3]. This provides the opportunity to directly detect the chiral asymmetry of the theory by observations [3–5]. Recently, in [6, 7] the above mentioned problem was addressed in the framework of

Hořava-Lifshitz (HL) theory of gravity [8], in which the Lorentz symmetry is broken in the ultraviolet (UV), and parity-violating operators generically appear. In particular, it was shown that, because of the parity violation and non-adiabatic evolution of the modes, a large polarization of PGWs can be produced, and could be well within the range of detection of the forthcoming CMB observations [7].

The effects of the parity violation on non-gaussianities of PGWs were also studied [9, 10] in the theory with the general covariance, and shown that, because of the symmetry of the pure de Sitter background, the parity violation from Weyl cubic terms have no contributions to the non-gaussianities, although this is no longer true when the coupling of Weyl cubic terms is time-dependent [11]. It should be noted that in all these studies the symmetry of the general diffeomorphisms of the underlaid theories plays an crucial role. On the other hand, in the HL theory the symmetry is reduced to the foliation-preserving diffeomorphisms [8], and the parity-violating operators allowed by such a symmetry are quite different from those with the general diffeomorphisms. Thus, it is expected that in the HL theory some distinguishable features of non-gaussianities of PGWs due to these parity-violating operators should exist, which may provide a smoking gun for the tests of the HL theory in the forthcoming CMB observations. With these motivations, in this paper we study the non-gaussianities of PGWs in the HL theory, and focus ourselves mainly on the effects of the high-order spatial operators on the non-gaussianities of PGWs, especially on the ones that violate the parity.

The rest of the paper is organized as follows: In Sec. II we first give a very brief review on the HL theory, and then restrict ourselves to the model recently proposed in [12, 13], where an extra $U(1)$ symmetry is enforced in the nonprojectable case, in order to eliminate the spin-0 gravitons usually appearing in the HL theory. In this sec-

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tion, we also present the linearized equation of motion of the tensor perturbations, originally derived in [7]. In Sec. III, from the cubic action of tensor perturbations, we calculate the three-point correlation function and obtain the bispectrum of the PGWs, while in Sec IV, we plot the shapes of bispectrum produced by both the second-order derivative operators and the three-dimensional parity-violating Chern-Simons one. In Sec V, we summarize our main results. There are also two appendices, A and B, in which the cubic action is given explicitly.

Before processing further, we would like to note that, although in this paper we restrict ourselves only to the model of the HL theory proposed recently in [12, 13], our results can be easily generalized to other models, as the tensor perturbations are quite similar in all of these models [7, 14]. In addition, non-gaussianities of PGWs in the framework of the general covariant theory with the projectability condition was also studied in [15], and several remarkable features were found. In particular, it was found that the terms $R_{ij}R^{ij}$ and $(\nabla^i R^{jk})(\nabla_i R_{jk})$ exhibit a peak at the squeezed limit, while the one $R_j^i R_k^j R_i^k$ favors the equilateral shape when spins of the three tensor fields are the same, but peaks in between the equilateral and squeezed limits when spins are mixed, where R_{ij} denotes the 3-dimensional Ricci tensor made of the 3-dimensional metric g_{ij} of the leaves $t = \text{Constant}$, and ∇_i denotes the covariant derivative with respect to g_{ij} . The consistency with the recently-released Planck observations [16] was also discussed. However, in [15] the parity-violating operators were excluded. Therefore, in this paper we shall focus mainly on the effects of these operators on non-gaussianities, as mentioned above. Moreover, non-gaussianities of scalar perturbations were also studied in the framework of the HL theory, one in the curvaton scenario [17] and the other in inflationary model [18], and some remarkable features were obtained.

II. NONPROJECTABLE GENERAL COVARIANT HL GRAVITY AND LINEAR TENSOR PERTURBATIONS

By construction, the HL theory is power-counting renormalizable [8]. This is achieved by breaking the symmetry of the general covariance in the UV, and include only high-order spatial derivative operators, so that it remains also unitary, a problem that has been facing for a long time in the quantization of gravity [19]. In the low energy, low dimensional operators take over, and it is expected that the Lorentz symmetry is “accidentally” restored [20]. Since Hořava first proposed it in 2009, the theory has attracted a lot of attention, partially because of various remarkable features when applied to cosmology [21], and partially because of some challenging questions, such as ghosts, instability and strong coupling. To overcome these questions, various models have been proposed [20], including the ones with an additional local $U(1)$ symmetry [12, 13, 22], in which the problems men-

tioned above can be avoided by properly choosing the coupling constants appearing in the theory. Since in all of those models, the tensor perturbations are almost the same [7, 14, 15], without loss of the generality, in this paper we shall work with the model proposed in [12, 13].

A. Action of the Nonprojectable General Covariant HL Gravity

The fundamental variables in the nonprojectable general covariant HL gravity proposed in [12, 13] are

$$(N, N^i, g_{ij}, A, \varphi),$$

where N and N^i denote, respectively, the lapse function and shift vector in the Arnowitt-Deser-Misner (ADM) decompositions [23], and A and φ are, respectively, the $U(1)$ gauge field and Newtonian prepotential [22]. Then, the corresponding total action can be cast in the form,

$$S = \zeta^2 \int dt d^3x \sqrt{g} N \left(\mathcal{L}_K - \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_\varphi + \zeta^{-2} \mathcal{L}_M \right), \quad (2.1)$$

where $\zeta^2 = 1/(16\pi G)$ with G being the Newtonian constant, \mathcal{L}_M describes matter fields, and

$$\begin{aligned} \mathcal{L}_K &= K_{ij}K^{ij} - \lambda K^2, \\ \mathcal{L}_V &= \mathcal{L}_V^R + \mathcal{L}_V^a, \\ \mathcal{L}_A &= \frac{A}{N} (2\Lambda_g - R), \\ \mathcal{L}_\varphi &= \varphi \mathcal{G}^{ij} (2K_{ij} + \nabla_i \nabla_j \varphi + a_i \nabla_j \varphi) \\ &\quad + (1 - \lambda) \left[(\Delta \varphi + a_i \nabla^i \varphi)^2 \right. \\ &\quad \left. + 2(\Delta \varphi + a_i \nabla^i \varphi) K \right] \\ &\quad + \frac{1}{3} \hat{\mathcal{G}}^{ijkl} \left[4(\nabla_i \nabla_j \varphi) a_{(k} \nabla_{l)} \varphi \right. \\ &\quad \left. + 5(a_i \nabla_j \varphi) a_{(k} \nabla_{l)} \varphi \right. \\ &\quad \left. + 2(\nabla_i \varphi) a_{j(k} \nabla_{l)} \varphi + 6K_{ij} a_{(l} \nabla_{k)} \varphi \right], \quad (2.2) \end{aligned}$$

with $\Delta \equiv \nabla^2$, and

$$\begin{aligned}
K_{ij} &= \frac{1}{2N} (-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i), \\
a_i &= \frac{N_{,i}}{N}, \quad a_{ij} = \nabla_j a_i, \\
\hat{\mathcal{G}}^{ijkl} &= g^{il} g^{jk} - g^{ij} g^{kl}, \\
\mathcal{G}_{ij} &= R_{ij} - \frac{1}{2} g_{ij} R + \Lambda_g g_{ij}, \\
\mathcal{L}_V^R &= \gamma_0 \zeta^2 + \gamma_1 R + \frac{\gamma_2 R^2 + \gamma_3 R_{ij} R^{ij}}{\zeta^2} + \frac{\gamma_5}{\zeta^4} C_{ij} C^{ij}, \\
\mathcal{L}_V^a &= -\beta_0 a_i a^i + \frac{1}{\zeta^2} \left[\beta_1 (a_i a^i)^2 + \beta_2 (a^i{}_i)^2 \right. \\
&\quad + \beta_3 (a_i a^i) a^j{}_j + \beta_4 a^{ij} a_{ij} + \beta_5 (a_i a^i) R \\
&\quad \left. + \beta_6 a_i a_j R^{ij} + \beta_7 R a^i{}_i \right] + \frac{1}{\zeta^4} \beta_8 (\Delta a^i)^2. \quad (2.3)
\end{aligned}$$

Here R denotes the Ricci scalar, and C_{ij} the Cotton tensor, defined by

$$C^{ij} = \frac{e^{ijk}}{\sqrt{g}} \nabla_k \left(R_l^j - \frac{1}{4} R \delta_l^j \right), \quad (2.4)$$

with $e^{123} = 1$, etc. $\lambda, \gamma_n, \beta_s$ and Λ_g are the coupling constants of the theory. In terms of R_{ij} , we have [13],

$$\begin{aligned}
C_{ij} C^{ij} &= \frac{1}{2} R^3 - \frac{5}{2} R R_{ij} R^{ij} + 3 R_j^i R_k^j R_i^k + \frac{3}{8} R \Delta R \\
&\quad + (\nabla_i R_{jk}) (\nabla^i R^{jk}) + \nabla_k G^k, \quad (2.5)
\end{aligned}$$

where

$$G^k = \frac{1}{2} R^{jk} \nabla_j R - R_{ij} \nabla^j R^{ik} - \frac{3}{8} R \nabla^k R. \quad (2.6)$$

It should be noted that in writing the above action, we have excluded all the terms that violate the parity [12, 13]. For our current purpose, we add the fifth and third-order spatial derivative operators to the potential \mathcal{L}_V [7],

$$\begin{aligned}
\Delta \mathcal{L}_V &= \frac{1}{M_*^3} (\alpha_0 K_{ij} R_{ij} + \alpha_2 \epsilon^{ijk} R_{il} \Delta_j R_k^l) \\
&\quad + \frac{\alpha_1 \omega_3(\Gamma)}{M_*} + \dots. \quad (2.7)
\end{aligned}$$

Here the coupling constant $\alpha_0, \alpha_1, \alpha_2$ are dimensionless and arbitrary, ϵ^{ijk} is the total antisymmetric tensor, and $\omega_3(\Gamma)$ the 3-dimensional gravitational Chern-Simons term¹. “...” denotes the rest of the fifth-order operators

given in Eq.(2.6) of [13]. Since they have no contributions to tensor perturbations, in this paper we shall not write them out explicitly. As shown in [7], because of the additional parity violation terms of Eq.(2.7), the non-adiabatic evolution of modes lead to a large polarization of PGWs, and it could be well within the detection of CMB observations, as mentioned above. In this paper, we investigate their effects on the non-gaussianities of PGWs.

B. The Linearized Tensor Perturbations

The general formulas of the linearized tensor perturbations were given in [7], so in the rest of this section we give a very brief summary of the main results obtained there, in order to initiate our studies of the non-gaussianities of PGWs in the next section. For details, we refer readers to [7]. Consider a flat Friedmann-Robertson-Walker (FRW) universe,

$$\begin{aligned}
\hat{N} &= a(\eta), \quad \hat{N}^i = \hat{A} = \hat{\varphi} = 0, \\
\hat{g}_{ij} dx^i dx^j &= a(\eta)^2 \delta_{ij} dx^i dx^j, \quad (2.8)
\end{aligned}$$

where quantities with hats denote the background of the FRW universe in the coordinates $(\eta, x^i) = (\eta, x, y, z)$. Then, the tensor perturbations are given by,

$$\begin{aligned}
\delta N &= \delta N^i = \delta A = \delta \varphi = 0, \\
\delta g_{ij} &= a^2 h_{ij}(\eta, \mathbf{x}). \quad (2.9)
\end{aligned}$$

Assuming that matter fields have no contributions to tensor perturbations, we find that the quadratic part of the total action can be cast in the form,

$$\begin{aligned}
S_g^{(2)} &= \zeta^2 \int d\eta d^3x \left\{ \frac{a^2}{4} (h'_{ij})^2 - \frac{1}{4} a^2 (\partial_k h_{ij})^2 \right. \\
&\quad - \frac{\hat{\gamma}_3}{4M_*^2} (\partial^2 h_{ij})^2 - \frac{\hat{\gamma}_5}{4M_*^4 a^2} (\partial^2 \partial_k h_{ij})^2 \\
&\quad - \frac{\alpha_1 a e^{ijk}}{2M_*} (\partial_l h_i^m \partial_m \partial_j h_k^l - \partial_l h_{im} \partial^l \partial_j h_k^m) \\
&\quad \left. - \frac{\alpha_2 e^{ijk}}{4M_*^3 a} (\partial^2 h_k^l)_{,j} - \frac{3\alpha_0 \mathcal{H}}{8M_* a} (\partial_k h_{ij})^2 \right\}, \quad (2.10)
\end{aligned}$$

where $h'_{ij} \equiv \partial h_{ij} / \partial \eta$, $\partial^2 \equiv \delta^{ij} \partial_i \partial_j$, $\mathcal{H} = a' / a$, and

$$\gamma_3 \equiv \left(\frac{M_{pl}}{2M_*} \right)^2 \hat{\gamma}_3, \quad \gamma_5 \equiv \left(\frac{M_{pl}}{2M_*} \right)^4 \hat{\gamma}_5.$$

To avoid fine-tuning, α_n and $\hat{\gamma}_n$ are expected to be of the same order. Then, the field equations for h_{ij} read,

$$\begin{aligned}
h''_{ij} &+ 2\mathcal{H} h'_{ij} - \alpha^2 \partial^2 h_{ij} + \frac{\hat{\gamma}_3}{a^2 M_*^2} \partial^4 h_{ij} - \frac{\hat{\gamma}_5}{a^4 M_*^4} \partial^6 h_{ij} \\
&+ e_i{}^{lk} \left(\frac{2\alpha_1}{M_* a} + \frac{\alpha_2}{M_*^3 a^3} \partial^2 \right) (\partial^2 h_{jk})_{,l} = 0, \quad (2.11)
\end{aligned}$$

¹ To take quantum effects into account, it was proposed to add boundary terms $\Delta S_3 = \sum_i \beta_i M^{3-\Delta_i} \int_{t=t_*} d^3x \sqrt{g} \mathcal{O}^i$ into the Einstein-Hilbert action at the moment $t = t_*$, right before the inflation started [24]. Clearly, one choice of \mathcal{O}^i is $\mathcal{O}^i \propto \omega(\Gamma)$. We thank Jiro Soda for pointing it out to us.

where $\alpha^2 \equiv 1 + 3\alpha_0 \mathcal{H}/(2M_*^3 a)$.

To study the evolution of h_{ij} , we expand it over spatial Fourier harmonics,

$$h_{ij}(\eta, \mathbf{x}) = \sum_{s=R,L} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \psi_k^s(\eta) e^{i\mathbf{k} \cdot \mathbf{x}} P_{ij}^{(s)}(\hat{\mathbf{k}}), \quad (2.12)$$

where $P_{ij}^{(s)}(\hat{\mathbf{k}})$ are the circular polarization tensors and satisfy the relations: $ik_m e^{rmj} P_{ij}^{(s)} = k \rho^s P_i^{r(s)}$ with $\rho^R = 1$, $\rho^L = -1$, and $P^{*i(s)} P_i^{j(s')} = \delta^{ss'}$ [6]. Define $u_k^s(\eta) = \frac{1}{2} a(\eta) M_{pl} \psi_k^s(\eta)$ and with the de Sitter background $a = -1/(H\eta)$, we obtain

$$u_k^s(\eta)'' + \left[\omega_s^2(k, \eta) - \frac{2}{\eta^2} \right] u_k^s(\eta) = 0, \quad (2.13)$$

where

$$\omega_s^2(k, \eta) \equiv \alpha^2 k^2 \left[1 - \delta_1 \rho^s (\epsilon_* \alpha k \eta) + \delta_2 (\epsilon_* \alpha k \eta)^2 + \delta_3 \rho^s (\epsilon_* \alpha k \eta)^3 + \delta_4 (\epsilon_* \alpha k \eta)^4 \right], \quad (2.14)$$

with $\epsilon_* \equiv H/M_* \ll 1$, and

$$\begin{aligned} \delta_1 &\equiv \frac{2\alpha_1}{\alpha^3}, & \delta_2 &\equiv \frac{\hat{\gamma}_3}{\alpha^4}, \\ \delta_3 &\equiv \frac{\alpha_2}{\alpha^5}, & \delta_4 &\equiv \frac{\hat{\gamma}_5}{\alpha^6}. \end{aligned} \quad (2.15)$$

Following [25, 26], we choose the initial conditions at $\eta = \eta_i$ as

$$u_k^s(\eta_i) = \frac{1}{\sqrt{2\omega_s(k, \eta_i)}}, \quad u_k^s(\eta_i)' = i \sqrt{\frac{\omega_s(k, \eta_i)}{2}}. \quad (2.16)$$

Then, if one assumes that $\omega_s(k, \eta)$ is slowly varying, i.e.,

$$\mathcal{Q} \equiv \left| \frac{\omega_s(k, \eta)'}{\omega_s^2(k, \eta)} \right| \ll 1, \quad (2.17)$$

one can approximatively treat $\omega_s(k, \eta)$ as constant, and get the approximate solution of the mode function $u_k^s(\eta)$,

$$u_k^s(\eta) \simeq \frac{1}{\sqrt{2\omega_s(k, \eta)}} \left(1 - \frac{i}{\omega_s \eta} \right) e^{-i \int \omega_s d\eta}, \quad (2.18)$$

or,

$$\psi_k^s(\eta) \simeq -2 \frac{iH}{M_{pl}} \frac{1}{\sqrt{2\omega_s^3}} (1 + i\omega_s \eta) e^{-i \int \omega_s d\eta}. \quad (2.19)$$

Promoting $\psi_k^s(\eta)$ to a quantum operator,

$$\psi_k^s(\eta) = \psi_k^s a_s(k) + \psi_{-k}^{*s} a_s^\dagger(-k), \quad (2.20)$$

one finds that the power spectrum of the tensor perturbations is given by

$$\Delta_{\text{T}}^2 \equiv \frac{k^3 (|\psi_k^R|^2 + |\psi_k^L|^2)}{2\pi^2} \simeq \frac{2H^2}{\pi^2 M_{pl}^2}, \quad (2.21)$$

which has the same expression as that given in general relativity. Here $a_s(k)$ and $a_s^\dagger(-k)$ are annihilation and creation operators, and their commutation relation is given by

$$[a_s(k), a_{s'}^\dagger(k')] = (2\pi)^3 \delta_{ss'} \delta(k - k').$$

It should be noted that, in our calculations we have assumed (2.17). This condition implies that the adiabatic condition is always satisfied (before the modes exit the horizon), and thus, there is no important modification in the power spectrum of PGWs. Once this condition is violated, as shown in [7], some interesting modifications on power spectrum and polarization of PGWs become possible. For simplification, in this paper we assume that (2.17) always holds.

III. THE INTERACTION HAMILTONIAN AND BISPECTRUM

In this section, we turn to the cubic action and the bispectrum of the tensor perturbations. The cubic action $S_g^{(3)}$ is given by Eq.(A.1), which can be written in the form,

$$S_g^{(3)} = - \int d\eta H_{\text{int}}(\eta). \quad (3.1)$$

Then the 3-point correlation function can be computed by employing the in-in formalism [27],

$$\begin{aligned} &\langle \psi_{k_1}^{s_1}(\eta) \psi_{k_2}^{s_2}(\eta) \psi_{k_3}^{s_3}(\eta) \rangle \\ &= -i \int_{\eta_i}^{\eta} d\eta' \langle [\psi_{k_1}^{s_1}(\eta) \psi_{k_2}^{s_2}(\eta) \psi_{k_3}^{s_3}(\eta), H_{\text{int}}(\eta')] \rangle, \end{aligned} \quad (3.2)$$

where η_i represents the early time when inflation starts, and η is a time when the bispectrum is evaluated. A good approximation is to extend the integral into the whole half axis, $\eta \in (-\infty, 0)$. After some simple but very tedious calculations, it can be shown that the 3-point correlation function can be rewritten in the form

$$\begin{aligned} &\langle \psi_{k_1}^{s_1}(0) \psi_{k_2}^{s_2}(0) \psi_{k_3}^{s_3}(0) \rangle \\ &= i(2\pi)^3 \delta^3(k_1 + k_2 + k_3) \zeta^2 \int_{-\infty}^0 a^2(\eta') d\eta' \\ &\quad \times F_{k_1 k_2 k_3}^{s_1 s_2 s_3}(\eta') \left[W_{k_1 k_2 k_3}^{s_1 s_2 s_3}(\eta') - W_{k_1 k_2 k_3}^{*s_1 s_2 s_3}(\eta') \right], \end{aligned} \quad (3.3)$$

where $F_{k_1 k_2 k_3}^{s_1 s_2 s_3}(\eta')$ is given in Appendix B, and $W_{k_1 k_2 k_3}^{s_1 s_2 s_3}(\eta')$ is defined as

$$\begin{aligned} W_{k_1 k_2 k_3}^{s_1 s_2 s_3}(\eta') &\equiv \psi_{k_1}^{s_1}(0) \psi_{k_2}^{s_2}(0) \psi_{k_3}^{s_3}(0) \\ &\quad \times \psi_{k_1}^{*s_1}(\eta') \psi_{k_2}^{*s_2}(\eta') \psi_{k_3}^{*s_3}(\eta'). \end{aligned} \quad (3.4)$$

In the de Sitter background, the 3-point correlation function reduces to,

$$\begin{aligned} & \langle \psi_{k_1}^{s_1}(0) \psi_{k_2}^{s_2}(0) \psi_{k_3}^{s_3}(0) \rangle \\ &= (2\pi)^7 \delta^3(k_1 + k_2 + k_3) \frac{\Delta_T^4}{2^3 k_1^3 k_2^3 k_3^3} B_{k_1, k_2, k_3}^{s_1 s_2 s_3}, \end{aligned} \quad (3.5)$$

where

$$B_{k_1 k_2 k_3}^{s_1 s_2 s_3} \equiv \sum_{n=0}^4 \delta_n \epsilon_*^n F_n I_n, \quad (3.6)$$

with $\delta_0 = 1$, and I_n is given by,

$$\begin{aligned} I_n \equiv & \text{Im} \left\{ \int_{-\infty}^0 d\eta (-\eta)^{n-2} e^{i \int (\omega_{s_1} + \omega_{s_2} + \omega_{s_3}) d\eta} \right. \\ & \times \sqrt{\frac{k_1^3 k_2^3 k_3^3}{\omega_{s_1}^3 \omega_{s_2}^3 \omega_{s_3}^3}} (1 - i\omega_{s_1} \eta) \\ & \left. \times (1 - i\omega_{s_2} \eta)(1 - i\omega_{s_3} \eta) \right\}. \end{aligned} \quad (3.7)$$

Usually, the k -dependence of the bispectrum receives contributions from both the interaction Hamiltonian $H_{\text{int}} (\sim \sum F_n)$ and the mode function integration I_n . For the former, one can see from (3.6) that the high order spatial derivative terms do have contributions in bispectrum, but are suppressed by the factor ϵ_* . Then, the leading-order contributions come from the two derivative term F_0 , which has the same expression as that given in general relativity.

On the other hand, the mode function integration I_n in the current case involved very complicated expression, and thus it is very hard to perform the integration explicitly. In general relativity, to minimize the errors, one usually splits the integrals into three different regions: one outside the horizon, one around the horizon, and one inside the horizon. The contribution from the last region vanishes due to the high-frequency oscillate [28]. A similar conclusion is also applicable to the current case, because when the high-order derivative terms dominate, the oscillation becomes more rapidly than that in general relativity. Thus, using the same arguments, one can safely neglect the effects from the high-order derivative terms in the last region.

In the first region, the mode is outside the horizon and the effective mass term $-2/\eta^2$ in Eq.(2.13) dominates. Then, in this region the corresponding results are also the same as those given in general relativity. Therefore, the effects from the parity-violating operators come only from the region around the horizon. In this region, although the k^2 term dominates, the high-derivative terms still have non-negligible contributions. In order to take these effects into account, we expand the integration in terms of ϵ_* . In particular, to the zeroth-order of ϵ_* , we

have

$$\begin{aligned} & \text{Im} \left\{ i \int_{-\infty}^0 d\eta (-\eta)^{n-2} e^{i(k_1 + k_2 + k_3)\eta} \right. \\ & \left. \times (1 + ik_1 \eta)(1 + ik_2 \eta)(1 + ik_3 \eta) \right\}, \end{aligned} \quad (3.8)$$

which coincides with the mode integration in general relativity, as expected. Thus, one immediately obtains the bispectrum of the leading-order,

$$\begin{aligned} B_{(\text{GR})}^{s_1 s_2 s_3}(k_1, k_2, k_3) = \\ \left(-K + \frac{k_1 k_2 + k_1 k_3 + k_2 k_3}{K} + \frac{k_1 k_2 k_3}{K} \right) F_0, \end{aligned} \quad (3.9)$$

where $K \equiv k_1 + k_2 + k_3$, and which is precisely the bispectrum of PGWs given in GR.

Now let us turn to the first order contributions of ϵ_* . Ignoring all the detailed calculations, it can be shown that it takes the form,

$$\begin{aligned} B_{(\text{PV})}^{s_1 s_2 s_3}(k_1, k_2, k_3) = \\ -\frac{\pi}{2} \delta_1 \epsilon_* \left[F_1 + \frac{3}{4} (s_1 k_1 + s_2 k_2 + s_3 k_3) F_0 \right]. \end{aligned} \quad (3.10)$$

Since this term is directly proportional to δ_1 , from Eqs.(2.7) and (2.15) we find that it represents the contributions of the three-dimensional Chern-Simons term.

For operators with dimensions $n \geq 4$, their contributions can be written in the form,

$$\epsilon_*^{n-2} \sum_r F_r \text{Im} \left\{ \int_{-\infty}^0 d\eta (-\eta)^{n-4} e^{iK\eta} f_{n-r-2}(\eta) \right\}, \quad (3.11)$$

where $r = (0, n-2)$, and $f_{n-r-2}(K, \eta)$ can be expressed as

$$\begin{aligned} f_{n-r-2}(K, \eta) \equiv & a_0 + a_1(i\eta) + a_2(i\eta)^2 + \dots \\ & + a_{n-r+1}(i\eta)^{n-r+1}, \end{aligned} \quad (3.12)$$

here a_r are functions of k_1, k_2, k_3 . In particular, the effects from fifth-order derivative terms should contribute to the bispectrum at the order of ϵ_*^3 . But, a careful analysis over the integration shows that their contributions vanish identically. This result can be easily generalized to higher order terms. In fact, for $n = 2j + 1$ with $j = 2, 3, 4, \dots$, the bispectrum of PGWs at the order ϵ_*^{n-2} always vanishes. This implies that $B_{(\text{PV})}^{s_1 s_2 s_3}(k_1, k_2, k_3)$ given by Eq.(3.10) represents the only contribution from parity violation operators.

In ref.[10], Soda et al calculated the bispectrum of PGWs from the Weyl cubic terms, W^3 and $\bar{W}W^2$, and proved that no contributions from parity violation appear in the non-gaussianity of PGWs in pure de Sitter

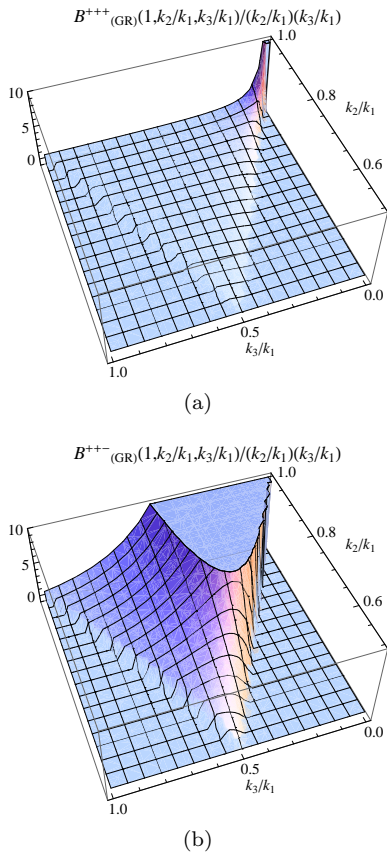


FIG. 1: Shapes of $(k_1 k_2 k_3)^{-1} B_{(\text{GR})}^{+++}(k_1, k_2, k_3)$ and $(k_1 k_2 k_3)^{-1} B_{(\text{GR})}^{++-}(k_1, k_2, k_3)$. All are normalized to unity in the equilateral limit.

background. This is consistent with our current results. However, it must be noted that in their considerations, the symmetry of the theory is still of the general diffeomorphisms. As a result, only the parity-violating terms W^3 and $\tilde{W}W^2$ are allowed. These terms are both P-odd and T-odd. Thus, when one calculates the bispectrum, the two terms produce an integral similar to (3.11) with $n - 2 = 2j - 1$ as odd number that is greater than two [10, 11]. Hence, with the arguments given above, they indeed have no contributions to the bispectrum of PGWs.

IV. SHAPE OF THE BISPECTRUM

We are now ready to plot the shapes of the bispectrum. For $s_1 = s_2 = s_3 = 1$ and $s_1 = s_2 = -s_3 = 1$, we plot the shapes of the bispectrum of the leading order contributions $(k_1 k_2 k_3)^{-1} B_{(\text{GR})}^{s_1 s_2 s_3}(k_1, k_2, k_3)$ in Fig. 1. Because there is no parity violation in the leading order, we have $B_{(\text{GR})}^{+++}(k_1, k_2, k_3) = B_{(\text{GR})}^{---}(k_1, k_2, k_3)$ and $B_{(\text{GR})}^{++-}(k_1, k_2, k_3) = B_{(\text{GR})}^{--+}(k_1, k_2, k_3)$. Thus, there are only two possible configurations, which both peak at the squeezed limit ($k_3/k_1 \rightarrow 0$). As pointed out in [29], the second configuration (Fig.1(b)) is sub-dominant,

in comparison with the first one in the equilateral limit ($k_1 \simeq k_2 \simeq k_3$), that is, $B_{(\text{GR})}^{++-} \simeq B_{(\text{GR})}^{+++}/81$.

Now we turn to the contributions from the parity-violating Chern-Simons term. In this case, because of the violation of the parity, for different spin products we have four independent configurations. We plot the shapes of these four configurations in Figs.(2), from which it can be seen that all the configurations peak in the squeezed limit (Note that for the $(++-)$ and $(---)$ cases they peak in the negative direction). More specifically, we have $B_{(\text{PV})}^{+++}(k_1, k_2, k_3) = -B_{(\text{PV})}^{---}(k_1, k_2, k_3)$, and $B_{(\text{PV})}^{++-}(k_1, k_2, k_3) = -B_{(\text{PV})}^{--+}(k_1, k_2, k_3)$.

V. CONCLUSIONS AND REMARKS

In this paper, we have investigated the non-gaussianities of PGWs generated during the de Sitter expansion of the universe in the framework of the HL theory, and paid particular attention on the effects of the operators that violate the parity. Because of the restricted foliation-preserving diffeomorphisms of the theory, the parity-violating third and fifth-dimensional operators exist generically. By calculating the three-point correlation function of the PGWs, we have shown that the leading-order contributions still come from the second-order spatial derivative terms, and are the same as those given in general relativity. The high-order n -th spatial derivative terms of the theory also contribute to the bispectrum, although their contributions are suppressed by a factor ϵ_*^{n-2} .

More remarkably, we have also found that the three-dimensional gravitational Chern-Simons operator $\omega(\Gamma)$ is the only one that violates the parity and meantime has non-vanishing contributions to the non-gaussianities of PGWs. In comparison with the contributions of the second-order operators that produce the same non-gaussianity as given in general relativity, its contributions are suppressed by the factor ϵ_* . In addition, operators with odd-order and higher than three have no contributions to the non-gaussianities of PGWs.

It should be noted that in obtaining the above results, we have assumed that the adiabatic condition (2.17) is always satisfied before the modes exit the horizon. If this condition fails to hold, the non-adiabatic evolution of the modes becomes possible, and hence the integral history of the mode function will be dramatically altered. Then, large non-gaussianities are expected, although it is still an open question how one can extract information of PGWs in this case.

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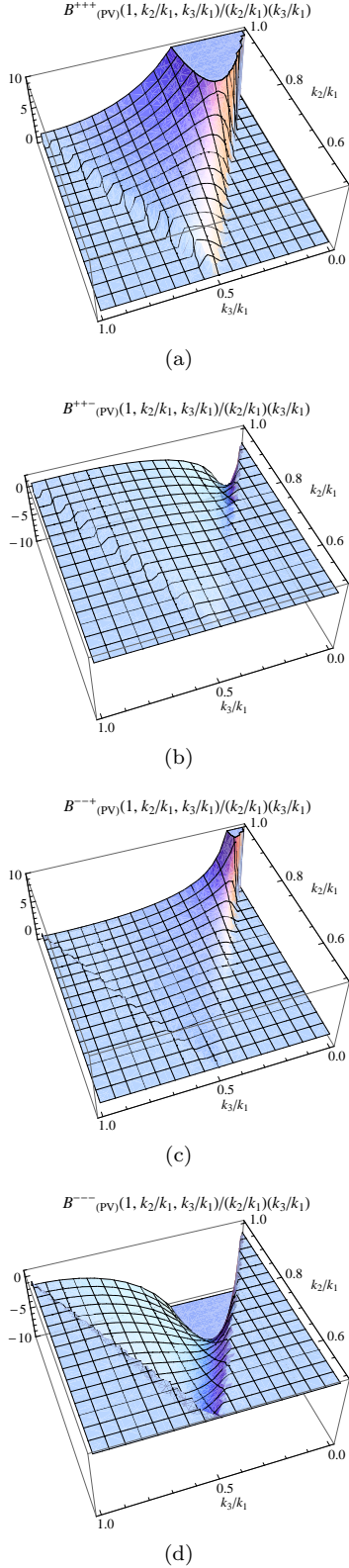


FIG. 2: Shapes of $(k_1 k_2 k_3)^{-1} B_{(PV)}^{s_1 s_2 s_3}(k_1, k_2, k_3)$ for various spin products: (a) $+++$; (b) $++-$; (c) $--+$; (d) $---$. All are normalized to unity in the equilateral limit.

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Appendix A: The Cubic Action of PGWs

The cubic action for the tensor perturbations can be written in the form,

$$S_g^{(3)} = \zeta^2 \int d\eta d^3 x a^2 \left\{ \frac{\alpha^2}{4} L_2 + \frac{\alpha_1}{a M_*} L_3 + \frac{\hat{\gamma}_3}{a^2 M_*^2} L_4 + \frac{\alpha_2}{a^3 M_*^3} L_5 + \frac{\hat{\gamma}_5}{a^4 M_*^4} L_6 \right\}, \quad (A.1)$$

where

$$\begin{aligned} L_2 &= h_{ij}^{mk} (2h_{mi} h_{kj} - h_{ij} h_{mk}), \\ L_3 &= -\frac{1}{2} e^{ijk} (\partial^2 h_{lk})_{,j} h_{im} h^{jm}, \\ &\quad -\frac{e^{ijk}}{4} \left[4h_{i[m,l]j} h^{lp} (h_{p,k}^m + h_{pk}^m - h_{k,p}^m) \right. \\ &\quad \left. + h_{i,l}^n h_{j,m}^l \left(\frac{2}{3} h_{k,n}^m - 2h_{kn}^m \right) \right. \\ &\quad \left. + 2h_{l,i}^n h_{m,j}^l h_{k,n}^m \right] \\ L_4 &= -\frac{1}{2} (\partial^2 h_{ij})_{,n}^m h_m^i h_n^j \\ &\quad -\frac{1}{2} (\partial^2 h_{ij}) h_{mn} \left(2h^{im,jn} - \frac{h^{mn,ij}}{2} - h^{ij,mn} \right), \\ L_5 &= \frac{e^{ijk}}{2} \left[-(\partial^2 h_{il}) h_{,j}^{mp} \left(h_{p,mk}^l - \frac{1}{2} h_{mp,k}^l \right) \right. \\ &\quad \left. + (\partial^2 h_{il,j}) h^{mp} (h_{mk,p}^l - h_{k,mp}^l) \right. \\ &\quad \left. + (\partial^2 h_{il,j}) h_{mk,p} 2h^{l[p,m]} \right. \\ &\quad \left. - \frac{1}{2} (\partial^2 h_{il,j}) h^{lp} \partial^2 h_{pk} \right] \\ &\quad -\frac{1}{8} e^{ijk} (\partial^2 h_{il}) (\partial^2 h_k^m) (h_{m,j}^l - h_{j,m}^l - h_{jm}^l) \\ &\quad + \frac{1}{4} e^{ijk} (\partial^4 h_{il,j}) h_{im} h^{lm}, \\ L_6 &= \frac{3}{8} (\partial^2 h_j^i) (\partial^2 h_k^j) (\partial^2 h_i^k) \\ &\quad + \frac{1}{2} \left[(\partial^4 h_{ij}) h_{mn} \left(2h^{im,jn} - \frac{1}{2} h^{mn,ij} - h^{ij,mn} \right) \right. \\ &\quad \left. + (\partial^4 h_{ij,n}) h_m^i h^{jn} + 2(\partial^2 h_{jk})_{,il} (\partial^2 h^{lk}) h^{ij} \right]. \end{aligned} \quad (A.2)$$

Then the interaction Hamiltonian is

where the even parity terms are

$$H_{\text{int}}(\eta) = \int d^3x \mathcal{H}_{\text{int}}(\eta, x),$$

$$\mathcal{H}_{\text{int}}(\eta, x) = -\zeta^2 a^2 \left[\frac{\alpha^2}{4} L_2 + \frac{\alpha_1}{a M_*} L_3 + \frac{\hat{\gamma}_3}{a^2 M_*^2} L_4 \right. \\ \left. + \frac{\alpha_2}{a^3 M_*^3} L_5 + \frac{\hat{\gamma}_5}{a^4 M_*^4} L_6 \right]. \quad (\text{A.3})$$

Appendix B: Expression of $F_{k_1 k_2 k_3}^{s_1 s_2 s_3}(\eta)$

$F_{k_1 k_2 k_3}^{s_1 s_2 s_3}(\eta)$ is given by,

$$F_{k_1 k_2 k_3}^{s_1 s_2 s_3}(\eta) = \alpha^2 F_0 + \frac{\alpha_1}{a M_*} F_1 + \frac{\hat{\gamma}_3}{a^2 M_*^2} F_2 \\ + \frac{\alpha_2}{a^3 M_*^3} F_3 + \frac{\hat{\gamma}_5}{a^4 M_*^4} F_4 \\ = \alpha^2 \sum_{n=0}^4 \delta_n \epsilon_*^n (-\eta)^n F_n, \quad (\text{B.1})$$

$$F_0 = \mathfrak{F}_k (k_1 s_1 + k_2 s_2 + k_3 s_3)^4, \\ F_2 = \mathfrak{F}_k (k_1 s_1 + k_2 s_2 + k_3 s_3)^4 \alpha^2 (k_1^2 + k_2^2 + k_3^2), \quad (\text{B.2})$$

$$F_4 = \alpha^4 \mathfrak{F}_k \left\{ k_1^8 + 4k_1^7 s_1 (k_2 s_2 + k_3 s_3) + 6k_1^6 (k_2 s_2 + k_3 s_3)^2 \right. \\ + 4k_1^5 s_1 (k_2^3 s_2 + 4k_2 k_3^2 s_2 + 4k_2^2 k_3 s_3 + k_3^3 s_3) \\ + k_1^4 (2k_2^4 + 19k_2^2 k_3^2 + 2k_3^4 + 16k_2 k_3 s_2 s_3 (k_2^2 + k_3^2)) \\ + 2k_1^3 s_1 (2k_2^5 s_2 + 8k_2^4 k_3 s_3 + 9k_2^3 k_3^2 s_2 + 9k_2^2 k_3^3 s_3 + 8k_2 k_3^4 s_2 + 2k_3^5 s_3) \\ + k_1^2 [6k_2^6 + 19k_2^4 k_3^2 + 19k_2^2 k_3^4 + 6k_3^6 + 2k_3 s_3 s_2 (8k_2^5 + 9k_2^3 k_3^2 + 8k_2 k_3^4)] \\ + 4k_1 s_1 (k_2^4 + k_2^2 k_3^2 + k_3^4) (k_2 s_2 + k_3 s_3)^3 \\ \left. + (k_2^4 + k_3^4) (k_2 s_2 + k_3 s_3)^4 \right\}, \quad (\text{B.3})$$

$$\mathfrak{F}_k \equiv -\frac{(k_1 - k_2 - k_3)(k_1 + k_2 - k_3)(k_1 - k_2 + k_3)(k_1 + k_2 + k_3)}{256k_1^2 k_2^2 k_3^2}, \quad (\text{B.4})$$

and the odd parity terms are

$$F_1 = -\alpha \mathfrak{F}_k \left\{ k_1^5 (4s_1 - 4s_2 - 5s_3) + k_2^5 (4s_2 - 4s_3 - 5s_1) + k_3^5 (4s_3 - 4s_1 - 5s_2) \right. \\ - 2k_1^3 k_2^2 (s_1 - 5s_2 - s_3 + 2s_1 s_2 s_3) - 2k_1^3 k_3^2 (s_1 - 4s_3 + 2s_1 s_2 s_3) \\ - 2k_2^3 k_3^2 (s_2 - 5s_3 - s_1 + 2s_1 s_2 s_3) - 2k_2^3 k_1^2 (s_2 - 4s_1 + 2s_1 s_2 s_3) \\ - 2k_3^3 k_1^2 (s_3 - 5s_1 - s_2 + 2s_1 s_2 s_3) - 2k_3^3 k_2^2 (s_3 - 4s_2 + 2s_1 s_2 s_3) \\ + k_1^4 k_3 (-6s_1 - 5s_2 + 6s_3 - 4s_1 s_2 s_3) + k_1^4 k_2 (-3s_1 + 6s_2 - 4s_3 - 4s_1 s_2 s_3) \\ + k_2^4 k_1 (-6s_2 - 5s_3 + 6s_1 - 4s_1 s_2 s_3) + k_2^4 k_3 (-3s_2 + 6s_3 - 4s_1 - 4s_1 s_2 s_3) \\ + k_3^4 k_2 (-6s_3 - 5s_1 + 6s_2 - 4s_1 s_2 s_3) + k_3^4 k_1 (-3s_3 + 6s_1 - 4s_2 - 4s_1 s_2 s_3) \\ + 2k_1^2 k_2^2 k_3 (5s_1 + 4s_2 - 2s_3 + 18s_1 s_2 s_3) + 2k_1^3 k_2 k_3 (-7s_1 + 7s_2 + 7s_3 + 2s_1 s_2 s_3) \\ + 2k_2^2 k_3^2 k_1 (5s_2 + 4s_3 - 2s_1 + 18s_1 s_2 s_3) + 2k_2^3 k_3 k_1 (-7s_2 + 7s_3 + 7s_1 + 2s_1 s_2 s_3) \\ \left. + 2k_3^2 k_1^2 k_2 (5s_3 + 4s_1 - 2s_2 + 18s_1 s_2 s_3) + 2k_3^3 k_1 k_2 (-7s_3 + 7s_1 + 7s_2 + 2s_1 s_2 s_3) \right\}, \quad (\text{B.5})$$

$$\begin{aligned}
F_3 = & \frac{\alpha^3}{2k_1^2 k_2^2 k_3^2} \mathfrak{F}_k \\
& \times \left\{ 2k_3^7 (k_2^3 - k_2 k_3^2)^2 s_1 + 2k_1^{11} k_3^2 s_2 \right. \\
& + 8k_1 k_2^3 (k_2 - k_3) k_3^7 (k_2 + k_3) s_2 \\
& - k_1^9 k_3^2 (4k_3^2 s_2 + 8k_2 k_3 s_3 + k_2^2 (-5s_1 + 8s_2 + 2s_3)) \\
& + k_1^7 k_3^2 (2k_3^4 s_2 + k_2^2 k_3^2 (-5s_1 + 20s_2 - 4s_3) \\
& + 8k_2 k_3^3 s_3 + 4k_2^3 k_3 s_2 s_3 (3s_1 + 4s_2 + 2s_3) \\
& + 2k_2^4 (-3s_1 + s_2 + 10s_3)) \\
& + k_1^8 k_2^2 k_3^2 (8k_3 s_1 (s_1 - s_3) s_3 \\
& + k_2 s_2 (1 + 8s_1 (s_1 - s_2 + s_3))) \\
& + k_1^3 k_2^2 k_3 s_1 (8k_2^6 k_3 s_2 (-s_1 + s_2) \\
& - 8k_2^7 + 4k_2 k_3^6 s_2 (4s_1 + 2s_2 + 3s_3) \\
& + 4k_2^5 k_3^2 s_3 (2s_1 + 3s_2 + 4s_3) \\
& + k_2^4 k_3^3 (8s_1 s_2 + 16s_2 s_3 - 3) \\
& + 8k_2^3 k_3^4 (2 - s_2 s_3 - s_1 (s_2 + s_3)) \\
& + k_3^7 (1 + 8s_3 (-s_1 + s_2 + s_3)) \\
& + 2k_2^2 k_3^5 (-1 + 2s_3 (2(s_1 + s_2) + s_3))) \\
& + k_1^4 k_2^2 (4k_2^3 k_3^4 (-s_1 + 4s_2) \\
& + 2k_3^7 (s_1 + 10s_2 - 3s_3) - 4k_2^7 s_3 \\
& + k_2^5 k_3^2 (-4s_1 - 5s_2 + 20s_3) \\
& + k_2 k_3^6 s_2 (-3 + 8(2s_1 + s_2) s_3) \\
& + 4k_2^2 k_3^5 (-s_2 + 4s_3) \\
& + 2k_2^4 k_3^3 s_3 (-1 + 2s_2 (2s_1 + s_2 + 2s_3))) \\
& + k_1^2 k_2^2 [k_2^7 k_3^2 (-2s_1 + 5s_2 - 8s_3) \\
& + k_2^2 k_3^7 (20s_1 - 4s_2 - 5s_3) + 2k_2^9 s_3 \\
& + k_2^6 k_3^3 (1 + 8s_2 (s_1 + s_2 - s_3)) s_3 \\
& + 2k_2^5 k_3^4 (10s_1 - 3s_2 + s_3) \\
& + 8k_2 k_3^8 s_2 s_3 (-s_2 + s_3) \\
& + k_3^9 (-8s_1 - 2s_2 + 5s_3) \\
& + k_2^4 k_3^5 (6s_2 - 9s_3 + 8s_2 s_3 (-2s_2 + s_3) \\
& + s_1 (-4 + 8s_2 s_3)) + k_2^3 k_3^6 (-2s_1 + 4s_3 + s_2 (-7 + 8(s_2 - 2s_3) s_3))] \\
& + k_1^5 k_2^2 k_3 [k_2^4 k_3 (4s_2 + s_1 (-7 + 8(s_1 - 2s_2) s_2) - 2s_3) \\
& + 8k_2^5 s_1 + 8k_2^3 k_3^2 s_3 (1 + s_2 (s_2 - s_3) - s_1 (s_2 + s_3)) \\
& + 8k_2 k_3^4 s_2 (1 + s_3 (-s_2 + s_3) - s_1 (s_2 + s_3)) \\
& + k_3^5 (-4s_2 + 6s_3 + s_1 (-9 + 8(s_1 + s_2 - 2s_3) s_3)) \\
& + 4k_2^2 k_3^3 (-s_3 + 4s_1)] \\
& + k_1^6 k_2^2 (2k_2^5 s_3 + k_2^2 k_3^3 s_3 (1 - 4 + 8s_1 (2s_2 + s_3)) \\
& + k_3^5 (-2s_2 - 7s_3 + 4s_1 (1 + 2s_3 (-2s_1 + s_3))) \\
& + 2k_2 k_3^4 s_2 (-1 + 2s_1 (s_1 + 2(s_2 + s_3))) \\
& + k_2^3 k_3^2 (-9s_2 - 4s_3 + 2s_1 (3 + 4s_2 (-2s_1 + s_2 + s_3))) \Big\}
\end{aligned}$$

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- [1] L.M. Krauss, S. Dodelson, and S. Meyer, *Science* **328**, 989 (2010); J. Garcia-Bellido, arXiv:1012.2006.
 - [2] U. Seljak and M. Zaldarriaga, *Phys. Rev. Lett.* **78**, 2054 (1997); M. Kamionkowski, A. Kosowsky, and A. Stebbins, *ibid.*, **78**, 2058 (1997); J. Bock et al., *Task Force on Cosmic Microwave Background Research*, arXiv:astro-ph/0604101.
 - [3] A. Lue, L. Wang, and M. Kamionkowski, *Phys. Rev. Lett.* **83**, 1506 (1999); N. Seto and A. Taruya, *ibid.*, **99**, 121101 (2007).
 - [4] S. Saito, K. Ichiki and A. Taruya, *JCAP*, **09**, 002 (2007); the Quiet Collaboration, *Astrophys. J.* **741**, 111 (2011).
 - [5] V. Gluscevic and M. Kamionkowski, *Phys. Rev. D* **81**, 123529 (2010).
 - [6] T. Takahashi and J. Soda, *Phys. Rev. Lett.* **102**, 231301 (2009).
 - [7] A. Wang, Q. Wu, W. Zhao, and T. Zhu, arXiv:1208.5490.
 - [8] P. Hořava, *Phys. Rev. D* **79**, 084008 (2009).
 - [9] J.M. Maldacena and G.L. Pimentel, *JHEP* **09**, 045 (2011).
 - [10] J. Soda, H. Kodama and M. Nozawa, *JHEP* **08**, 067 (2011).
 - [11] M. Shiraishi, D. Nitta, and S. Yokoyama, *Prog. Theor. Phys.* **126**, 937 (2011).
 - [12] T. Zhu, Q. Wu, A. Wang, and F.-W. Shu, *Phys. Rev. D* **84**, 101502 (R) (2011).
 - [13] T. Zhu, F.-W. Shu, Q. Wu, and A. Wang, *Phys. Rev. D* **85**, 044053 (2012).
 - [14] A. Wang, *Phys. Rev. D* **82**, 124063 (2010).
 - [15] Y.-Q. Huang, A. Wang, R. Yousefi, and T. Zhu, arXiv:1304.1556.
 - [16] Planck Collaboration, arXiv:1303.5076.
 - [17] K. Izumi, T. Kobayashi, and S. Mukohyama, *J. Cosmol. Astropart. Phys.* **10** (2010) 031.
 - [18] Y.-Q. Huang and A. Wang, *Phys. Rev. D* **86**, 103523 (2012).
 - [19] K.S. Stelle, *Phys. Rev. D* **16**, 953 (1977).
 - [20] D. Blas, O. Pujolas, and S. Sibiryakov, *JHEP* **1104**, 018 (2011); P. Hořava, *Class. Quantum Grav.* **28**, 114012 (2011); T. Clifton, P.G. Ferreira, A. Padilla, and C. Skordis, *Phys. Rep.* **513**, 1 (2012).
 - [21] S. Mukohyama, *Class. Quantum Grav.* **27**, 223101 (2010).
 - [22] P. Hořava and C.M. Melby-Thompson, *Phys. Rev. D* **82**, 064027 (2010); A. Wang and Y. Wu, *Phys. Rev. D* **83**, 044031 (2011); A.M. da Silva, *Class. Quan. Grav.* **28**, 055011 (2011).
 - [23] R. Arnowitt, S. Deser, and C.W. Misner, *Gen. Relativ. Grav.* **40**, 1997 (2008); C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation* (W.H. Freeman and Company, San Francisco, 1973), pp.484-528.
 - [24] M. Porrati, arXiv:hep-th/0409210.
 - [25] J. Martin and R. Brandenberger, *Phys. Rev. D* **63**, 123501 (2001).
 - [26] J. Martin and R. Brandenberger, *Phys. Rev. D* **68**, 063513 (2003); R. Brandenberger and J. Martin, *Phys. Rev. D* **71**, 023504 (2005).
 - [27] J. Maldacena, *JHEP*, **05**, 013 (2003); S. Weinberg, *Phys. Rev. D* **72**, 043514 (2005).
 - [28] J. Maldacena, *JHEP* **05** (2003) 013.
 - [29] X. Gao, *et al.*, *Phys. Rev. Lett.* **107**, 211301 (2011).